This article was downloaded by:
On: *25 January 2011*
Access details: *Access Details: Free Access*
Publisher *Taylor & Francis*
Informa Ltd Registered in England and Wales Registered Number: 1072954 Registered office: Mortimer House, 37-41 Mortimer Street, London W1T 3JH, UK

Liquid Crystals

Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713926090

On the response speed of pi-cells

H. G. Walton[a]; M. J. Towler[a]
[a] Sharp Laboratories of Europe Ltd, Oxford Science Park, Oxford OX4 4GA, UK,

Online publication date: 06 August 2010

To cite this Article Walton, H. G. and Towler, M. J.(2000) 'On the response speed of pi-cells', Liquid Crystals, 27: 10, 1329 — 1335
To link to this Article: DOI: 10.1080/026782900423386
URL: http://dx.doi.org/10.1080/026782900423386

PLEASE SCROLL DOWN FOR ARTICLE

Full terms and conditions of use: http://www.informaworld.com/terms-and-conditions-of-access.pdf

This article may be used for research, teaching and private study purposes. Any substantial or systematic reproduction, re-distribution, re-selling, loan or sub-licensing, systematic supply or distribution in any form to anyone is expressly forbidden.

The publisher does not give any warranty express or implied or make any representation that the contents will be complete or accurate or up to date. The accuracy of any instructions, formulae and drug doses should be independently verified with primary sources. The publisher shall not be liable for any loss, actions, claims, proceedings, demand or costs or damages whatsoever or howsoever caused arising directly or indirectly in connection with or arising out of the use of this material.

LIQUID CRYSTALS, 2000, VOL. 27, NO. 10, 1329–1335

On the response speed of pi-cells

H. G. WALTON* and M. J. TOWLER
Sharp Laboratories of Europe Ltd, Oxford Science Park,
Oxford OX4 4GA, UK

(*Received 16 February 2000; accepted 14 April 2000*)

We discuss the role of the Miesowicz viscosities on director reorientation in pi-cells, showing that the symmetry of this system allows for a simplification of the dynamical equations. We consider practical aspects of material optimisation for increased pi-cell switching speed. We make an observation regarding the switching speeds of positive and negative dielectric anisotropy surface mode devices.

1. Introduction

The pi-cell [1] is a nematic liquid crystal device which can readily be made to exhibit combined 'on + off' response times of less than 10 ms, and is thus an attractive candidate for video-rate displays. It comprises a thin (typically ~ 1–10 μm) slab of liquid crystal (thickness d) confined between two rubbed polymer surfaces, arranged with rubbing directions parallel, giving equal and oppositely directed pretilt (θ_p) at the two surfaces, i.e. $\theta_p(z = 0) = -\theta_p(z = d)$ (see figure 1). At zero volts the pi-cell adopts an untwisted splayed configuration, with the director at the cell centre lying parallel to the cell surfaces. Above some critical voltage however, typically ~ 2 V for conventional display materials, a bend director-configuration becomes the favoured minimum energy state, with the director at the cell centre aligning along the electric field, perpendicular to the cell surfaces. With the system in the bend state, a modulation of the applied voltage causes director distortion only in the vicinity of the cell surfaces, with the cell centre remaining permanently aligned along the field. During switching, the two surface regions of a pi-cell are thus decoupled, and it is this feature which ultimately leads to its rapid electro-optic characteristics.

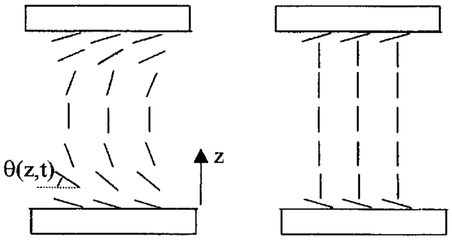


Figure 1. Schematic illustration of a pi-cell in the low voltage (left) and high voltage (right) bend state.

Bos *et al.* [1] correctly related the rapid response of the pi-cell to the lack of backflow during director reorientation. The consequences of the pi-cell's symmetry for director motion do not appear to have been fully elucidated in the literature however. In this paper we describe how the symmetry of the pi-cell allows for certain simplifications when calculating director dynamics. We also consider the role of the material viscosity on switching dynamics.

2. Equations of motion and analysis

The general visco-elastic equations of motion for a nematic liquid crystal have been developed by Ericksen [2] and Leslie [3]. We consider an untwisted slab of infinite extent in the xy plane and finite thickness, d, along z. We consider only variations of velocity along z, taking the one elastic constant ($K_{11} = K_{33} \equiv K$) approximation. We ignore inertial terms. Under these approximations, the equations of motion describing director switching reduce to two coupled partial differential equations in terms of the fluid velocity, $v(z, t)$, and the director tilt angle, $\theta(z, t)$ (we choose $\theta = 0°$ to be in the plane of the surfaces):

$$C(t) = f(\eta, \theta)\frac{\partial \theta}{\partial t} + g(\eta, \theta)\frac{\partial v(z, t)}{\partial z} \tag{1}$$

$$K\frac{\partial^2 \theta(z, t)}{\partial^2 z} + \Delta\varepsilon\varepsilon_0 E^2 \sin\theta(z, t)\cos\theta(z, t) = \gamma_1 \frac{\partial \theta}{\partial t} + f(\eta, \theta)\frac{\partial v(z, t)}{\partial z} \tag{2}$$

where γ_1 is the nematic rotational viscosity and the functions f and g depend upon θ and the Miesowicz viscosities [4, 5] η_1, η_2 and η_{12} which describe, respectively: viscosity for the director in the shear plane and parallel to the velocity gradient; in the shear plane and parallel

---

*Author for correspondence; e-mail: Harryw@sharp.co.uk

Liquid Crystals ISSN 0267-8292 print/ISSN 1366-5855 online © 2000 Taylor & Francis Ltd
http://www.tandf.co.uk/journals

to the shear direction; and for an irrotational flow. (For ease of reference, Appendix I lists the relationships between the Meisowicz and the Leslie coefficients, α_i). Explicitly, f and g are given by:

$$f = \left(\frac{\gamma_1 - \eta_1 + \eta_2}{2}\right)\cos^2\theta + \left(\frac{\gamma_1 + \eta_1 - \eta_2}{2}\right)\sin^2\theta \quad (3)$$

$$g = \eta_{12}\sin^2\theta\cos^2\theta + (\eta_1 - \eta_2)\sin^2\theta + \eta_2. \quad (4)$$

The one-dimensional dynamical behaviour of a nematic layer may be found by solving equations (1) and (2), subject to appropriate boundary conditions, to find $v(z, t)$ and $\theta(z, t)$. A general approach to obtaining numerical solutions for nematics has been described elsewhere [6, 7]; it can be briefly summarized as follows. Firstly, one seeks a value for C (a function of t but not z) in equation (1). Integrating with respect to z we note that the second term on the right vanishes since, by the no-slip boundary condition, $v(0, t) = v(d, t) = 0$. To integrate the term $f\partial\theta/\partial t$ with respect to z, we make an initial guess for $\partial\theta(z)/\partial t$, and integrate numerically to find C. Knowing C we may eliminate $v(z, t)$ from equation (2). Given an initial director profile $\theta(z, 0)$ (e.g., from a statics calculation), we can now integrate equation (2) forward to obtain a first approximation of $\theta(z, \Delta t)$. Next $\partial\theta(z)/\partial t$ is re-calculated, substituted back into equation (1), and the process repeated until convergence is obtained. Finally, the whole process can be repeated, stepping forward in time. At each time-step one is normally required to recalculate C.

This procedure is appropriate to arbitrary one-dimensional, untwisted nematic layers. In the case of the pi-cell however, symmetry allows simplification. We note that C is identically zero at all times, since the functions f, g are symmetric and $\partial\theta/\partial t$, $\partial v/\partial z$ are anti-symmetric about the cell mid-plane whence the integrals of $f\partial\theta/\partial t$ and $g\partial v/\partial z$ with respect to z evaluate to zero. We can therefore set $C = 0$ and immediately substitute for $v(z, t)$ in equation (2). We also choose to rewrite E in terms of the displacement field D, which is constant along z in the absence of free charges. D is related to the applied voltage, V, through:

$$D = -V\bigg/\int_0^d \frac{\mathrm{d}z}{\varepsilon_0(\varepsilon_\parallel \sin^2\theta + \varepsilon_\perp \cos^2\theta)} \quad (5)$$

With these substitutions, equations (1) and (2) simplify to a single non-linear diffusion equation (6) appropriate to the description of pi-cell dynamics:

$$\frac{\partial^2\theta}{\partial Z^2} = \frac{\partial\theta}{\partial T}[1 - \gamma^*(\theta)] + D'^2\frac{\cos\theta\sin\theta}{(\varepsilon_\parallel \sin^2\theta + \varepsilon_\perp \cos^2\theta)} \quad (6)$$

where we have normalized through a change in coordinates $z \to Zd$, $t \to Td^2\gamma_1/K$ and $D \to D'(K\varepsilon_0/\Delta\varepsilon d^2)^{1/2}$.

The function $\gamma^*(\theta)$ is given explicitly by:

$$\gamma^* = \frac{\left[\frac{1}{2}\left(1 - \frac{\eta_1 - \eta_2}{\gamma_1}\right)\cos^2\theta + \frac{1}{2}\left(1 + \frac{\eta_1 - \eta_2}{\gamma_1}\right)\sin^2\theta\right]^2}{\frac{\eta_{12}}{\gamma_1}\sin^2\theta\cos^2\theta + \left(\frac{\eta_1 - \eta_2}{\gamma_1}\right)\sin^2\theta + \frac{\eta_2}{\gamma_1}}. \quad (7)$$

The term $(1 - \gamma^*)$ in equation (6) has the role of a (normalized) rotational viscosity and we see that the effect of including flow in the equations of motion is equivalent to modifying the rotational viscosity by an amount γ^*. A similar result is obtained [8] when considering the influence of backflow on the dynamics of the Fréedericksz cell, but there the value of the constant C in equation (1) is time dependant and interpretation is less transparent.

Equation (6) is in a convenient form for numerical analysis, but before proceeding we can gain insight into pi-cell dynamics by making some approximations. We set $\eta_1/\gamma_1 = 1$, $\eta_{12}/\gamma_1 = 0$, and $\eta_2/\gamma_1 = 0$. Relatively few experimental reports exist giving full sets of measured viscosities for nematics. However, when comparing our approximations with the measured values for MBBA [9], for which $\eta_1 = 121$, $\eta_2 = 23.8$, $\eta_{12} = 6.5$, $\gamma_1 = 95$ $(\times 10^{-3}\,\mathrm{kg}^{-1}\,\mathrm{s}^{-1})$, we can regard the first two approximations as reasonable. The third is clearly less easily justified; nevertheless it allows us to gain some insight. In addition we set $D' = 0$ which corresponds to a situation of director relaxation following removal of an applied voltage. With these simplifications, equation (6) reduces to:

$$\frac{\partial^2\theta}{\partial Z^2} = \cos^2\theta\frac{\partial\theta}{\partial T}. \quad (8)$$

We see that for the stated approximations, the symmetric nature of the pi-cell modifies by a factor $\cos^2\theta$, the well known 1D-diffusion equation:

$$\partial^2\theta/\partial Z^2 = \partial\theta/\partial T \quad (9)$$

often employed as a simple model for director relaxation in nematic layers when flow is ignored. Since $\cos^2\theta \leqslant 1$, we expect the solution corresponding to the tilt relaxation rate from equation (8) to lead (i.e. be faster than) that of equation (9). Mathematically this may be proved (see Appendix II) and we conclude that flow-symmetry in the pi-cell increases relaxation speed.

Figure 2(*a*) shows a plot of director relaxation obtained by solving equation (8). The cell relaxes from an initial $(T = 0)$ high voltage bend state towards an

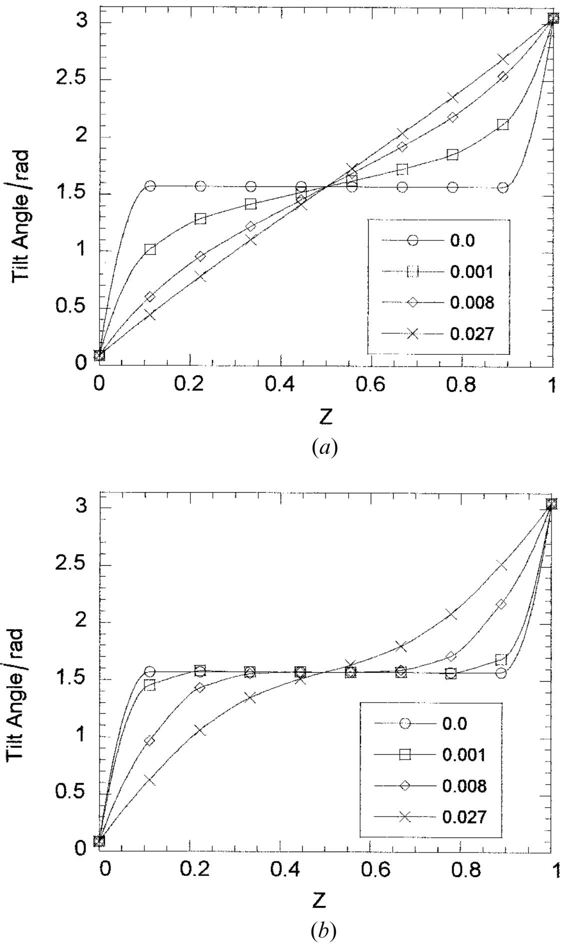


Figure 2. (*a*) Pi-cell relaxation (8 V to 0 V) based on model of equation (8). (*b*) Pi-cell relaxation (8 V to 0 V) ignoring flow contribution to dynamics, equation (9).

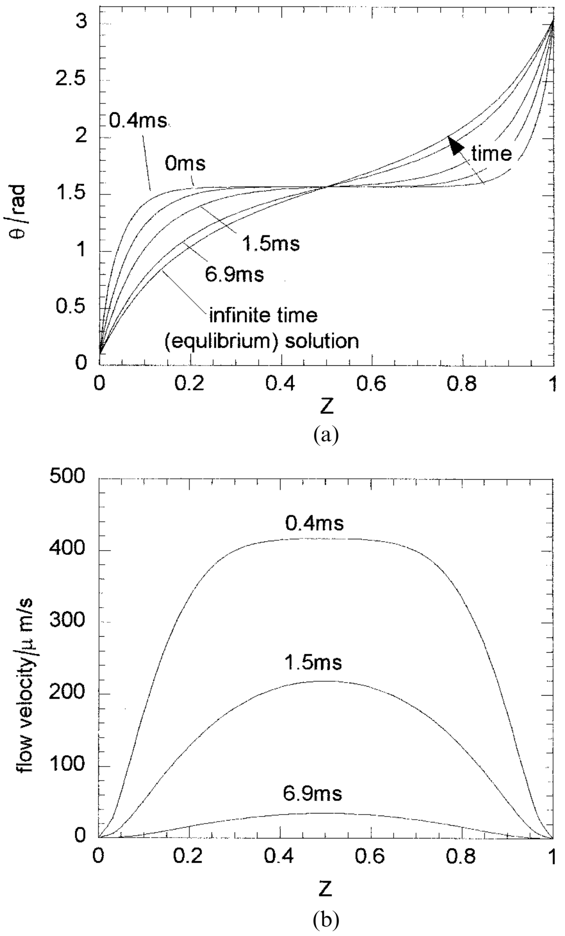


Figure 3. (*a*) Director tilt angle relaxation in ZLI-2293 pi-cell, following 8 V to 2 V switch. (*b*) Flow velocity profile of device in figure 3 (*a*) as a function of time.

equilibrium zero volt bend state†. The consequence of ignoring flow—i.e. equation (9)—is shown in figure 2 (*b*). Clearly, a slower response results (comparing for example, the two director configurations at dimensionless time $T = 0.001$), highlighting the role of flow in pi-cell relaxation.

Returning now to the full equation of motion (6), figures 3 (*a*) and 3 (*b*) show a numerical simulation of the commercial nematic ZLI-2293 (Merck) relaxing from 8 V to 2 V in a 5°-pretilt, $d = 7.5$ μm, pi-cell. Viscosity coefficients were taken from [10]. Numerical solutions were obtained by the procedure previously outlined, noting additionally that D' must be re-calculated at each time step using equation (5).

For completeness we note that some caution is necessary when calculating velocity profiles such as figure 3 (*b*). Our initial approximation that the liquid crystal moment of inertia is zero, allows for the non-physical appearance of large velocities at arbitrarily small times. Following van Doorn [6] however, we can hope that inertial effects are only important over an initial time scale $\tau \sim$ mass/damping $= d^2\rho/\gamma_1 \sim 1$ μs (where ρ is the LC density) and thus we can ignore them here. Retaining inertial terms in the equations of motion amounts to retaining terms in $\partial^2\theta/\partial t^2$. We have not attempted to solve this system of equations.

3. Effect of viscosity parameters on electro-optic response

We now make some observations regarding the role of the nematic viscosities in pi-cell dynamics. Firstly we recall that we do not have complete freedom, even in principle, to vary material viscosities. Thermodynamic arguments [3] establish certain relationships, for our

---

† In real pi-cells, the 0 V bend state is of course unstable with respect to a splay state director configuration. The transition between these two states however involves the nucleation of defects and is typically rather slow (~ secs). It is therefore acceptable in our calculation to treat the bend state as metastable at 0 V.

purposes the most important being:

$$\gamma_1(2\eta_1 + 2\eta_2 - \gamma_1) > (\eta_2 - \eta_1)^2 \qquad (10)$$

and

$$\eta_{12} + 2(\eta_1 + \eta_2) - \gamma_1 - \eta_3 > 0 \qquad (11)$$

together with the constraint that γ_1, η_1, η_2, η_3 must be positive (η_{12} can be positive or negative). η_3 is the viscosity coefficient for the director lying normal to the shear plane, and behaves in many respects as a 'background' or isotropic viscosity. Equations (10) and (11) ensure $(1 - \gamma^*) > 0$ in equation (6).

To analyse the role of the viscosity on pi-cell switching, equation (7) indicates that we consider the influence of the three terms η_{12}/γ_1, η_2/γ_1 and $(\eta_1 - \eta_2)/\gamma_1$ on the overall viscosity, $(1 - \gamma^*)$ appearing in equation (6). (Of course, the value of $(1 - \gamma^*)$ is also a function of director tilt.) Fixing $(\eta_1 - \eta_2)/\gamma_1 = 1$, $\eta_2/\gamma_1 = 0.25$ and plotting $(1 - \gamma^*)$ against η_{12} (see figure 4) reveals that variation of η_{12} is of relatively small consequence for the overall viscosity $(1 - \gamma^*)$, with the greatest influence seen for the director lying at the 45° tilt position.

Considering next the parameter η_2/γ_1, figure 5(*a*) shows the effect of fixing $(\eta_1 - \eta_2)/\gamma_1 = 1$, $\eta_{12}/\gamma_1 = 0.1$ and varying η_2/γ_1 between 0 to 1. As seen previously, equation (8), when $\eta_2/\gamma_1 = 0$ the overall viscosity, $(1 - \gamma^*)$, reduces to a $\cos^2\theta$ variation. For calamitic molecules, $\eta_1 > \eta_2$ and we obtain the solid lines in figure 4. For completeness, and in connection with a device to be discussed later, the broken lines indicate the effect of taking $\eta_2 > \eta_1$ (equivalent to considering discotic molecules), fixing $(\eta_2 - \eta_1)/\gamma_1 = 1$, $\eta_{12}/\gamma_1 = 0$ and varying η_2/γ_1 from 1 to 2. In this case $(1 - \gamma^*)$ varies in a $\sin^2\theta$-like fashion.

Figure 5(*b*) shows the modelled optical response (for 550 nm light) of a 7 μm, 5°-pretilt pi-cell, using a material (again based on ZLI-2293) with dielectric constants $\varepsilon_\parallel = 14.1$, $\varepsilon_\perp = 4.1$, refractive indices $n_o = 1.4895$, $n_e = 1.6122$ and $K_{11} = K_{33} = 10$ pN. The device is shown relaxing from 8 V to 2 V, held at 45° between crossed polarizers. In the optical modelling an additional static 45 nm waveplate is present to ensure that optical extinction occurs at 8 V. The parameters $(\eta_1 - \eta_2)/\gamma_1 = 1$, $\eta_{12}/\gamma_1 = 0.1$ are fixed and η_2/γ_1 is varied.

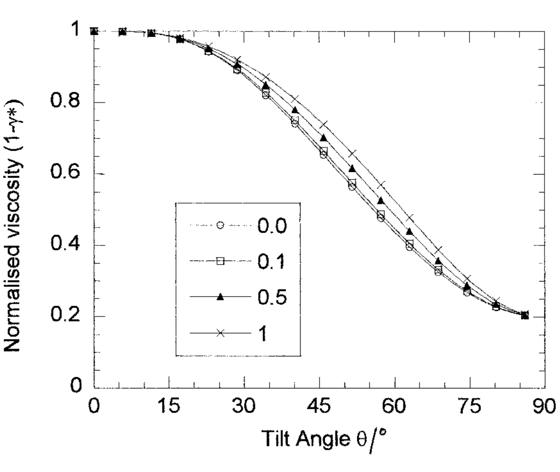
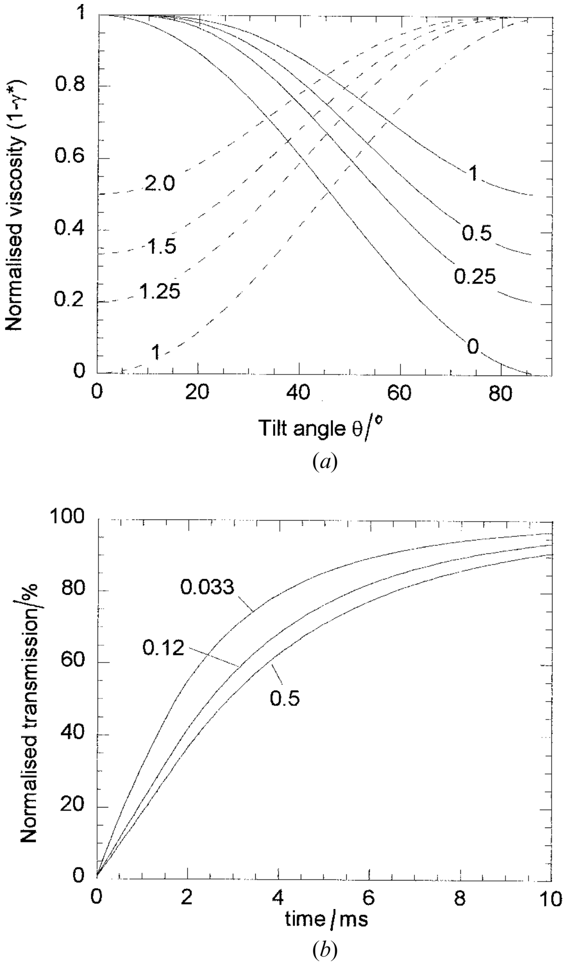


Figure 4. Effect of η_{12} variation (values in inset) on normalized pi-cell viscosity, $1 - \gamma^*(\theta)$. We constrain $(\eta_1 - \eta_2)/\gamma_1 = 1$ and $\eta_2/\gamma_1 = 0.25$.



(*a*)

(*b*)

Figure 5. (*a*) Effect of η_2/γ_1 variation (values given on graph) on normalized pi-cell viscosity, $1 - \gamma^*(\theta)$; we constrain $\eta_{12} = 0.1$. Solid lines indicate $\eta_1 > \eta_2$ with constraint $(\eta_1 - \eta_2)/\gamma_1 = 1$, and broken lines indicate $\eta_2 > \eta_1$ with $(\eta_2 - \eta_1)/\gamma_1 = 1$. (*b*) Effect of η_2/γ_1 variation (values given on graph) on optical transmission of pi-cell following 8 V to 2 V switch. We constrain $\eta_{12} = 0$ and $(\eta_1 - \eta_2)/\gamma_1 = 1$.

Turning lastly to the influence of the parameter $(\eta_1 - \eta_2)/\gamma_1$ on the viscosity $(1 - \gamma^*)$. In figure 6(*a*), $\eta_2/\gamma_1 = 0.25$, $\eta_{12}/\gamma_1 = 0.1$ are held fixed and $(\eta_1 - \eta_2)/\gamma_1$ varied. It is apparent that a real material with $(\eta_1 - \eta_2)/\gamma_1 \sim 0$ would show a low normalized viscosity for the director moving through any angle. It is interesting to note that the normalized viscosity for switching through certain limited angular ranges can be lowered by choosing

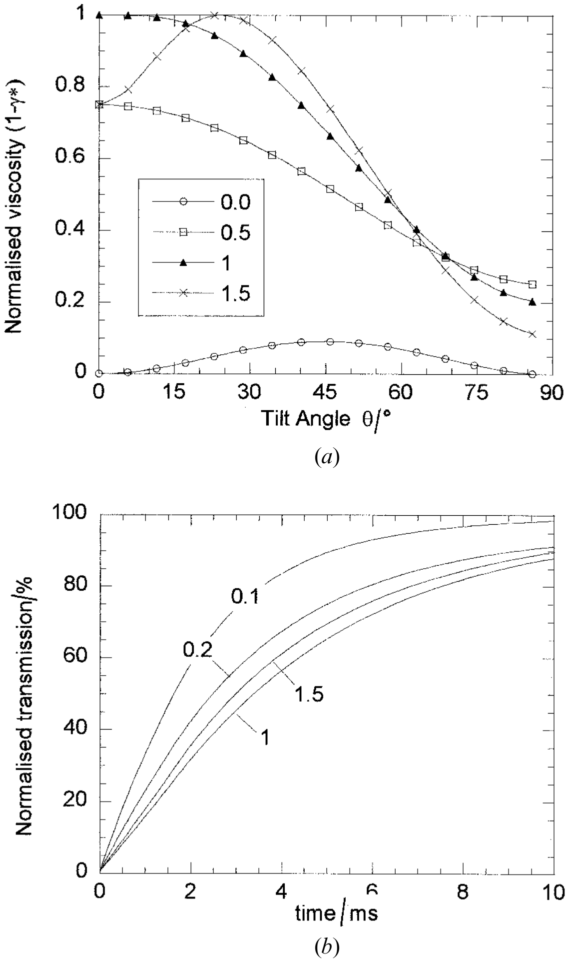


Figure 6. (*a*) Effect of $(\eta_1-\eta_2)/\gamma_1$ variation (values in inset) on normalized pi-cell viscosity, $1-\gamma^*(\theta)$. We constrain $\eta_{12}=0.1$ and $\eta_2/\gamma_1=0.25$. (*b*) Effect of $(\eta_1-\eta_2)/\gamma_1$ variation (values given on graph) on optical transmission of pi-cell following 8 V to 2 V switch. We constrain $\eta_{12}=0.1$ and $\eta_2/\gamma_1=0.25$. (All curves asymptote to ~ 100% transmission value, having largely converged by time ~ 30 ms).

large finite values for $(\eta_1-\eta_2)/\gamma_1$. For example, taking $(\eta_1-\eta_2)/\gamma_1=1.5$ in figure 6(*a*) especially reduces the viscosity for molecules reorienting through angles in the range ~60–90°. One might be tempted to try to exploit this feature by taking still more extreme values; $\eta_2/\gamma_1=0.25$ and $(\eta_1-\eta_2)/\gamma_1=2.5$ for example. However, this then violates the constraint of equation (10), resulting in (non-physical) negative values for $(1-\gamma^*)$.

Figure 6(*b*) shows the variation in optical relaxation time as $(\eta_1-\eta_2)/\gamma_1$ is varied, with $\eta_2/\gamma_1=0.25$, $\eta_{12}/\gamma_1=0.1$ held fixed. Other cell and material parameters are as stated above in connection with figure 5(*b*).

4. Practical aspects of materials design for pi-cells

It is of interest to consider what practical scope there is for attempting to optimize the viscosity parameters of real materials. A complete set of viscosity coefficients appears to have been determined for only a limited number of materials, and the scope for tuning them seems largely unclear. Qualitative rules for the design of materials [11] appear to offer opportunity for systematic variation of γ_1 and η_2 (the latter relates to the so-called 'bulk viscosity' sometimes quoted for nematic flow down a capilliary tube with the director parallel to the flow). The material factors affecting η_{12} do not appear to be understood although, as shown previously, this parameter seems to be of relatively minor importance in determining electro-optic behaviour.

Figures 6(*a*) and 6(*b*) indicate that an ability to vary $(\eta_1-\eta_2)/\gamma_1$ might have interesting consequences. When reviewing the literature we find in practice however, that this parameter very commonly has a value close to unity (for example, all six materials in [10] have $(\eta_1-\eta_2)/\gamma_1 \sim 1.0 \pm 0.1$ across a range of temperatures). In the Leslie notation:

$$\frac{\eta_1-\eta_2}{\gamma_1} \equiv \frac{-1-(\alpha_3/\alpha_2)}{-1+(\alpha_3/\alpha_2)}$$

where the viscosity coefficient α_2 is negative, whilst α_3 may be positive or negative. The practical task of having $(\eta_1-\eta_2)/\gamma_1$ differing appreciably from unity can then be restated as the requirement to have $|\alpha_3/\alpha_2| \sim 1$. Again however, experimentally one commonly finds $\alpha_3 \ll \alpha_2$ (for example, $\alpha_3=-0.0011$ Pa s, $\alpha_2=-0.11$ Pa s at 25°C for MBBA [12]). Furthermore, microscopic theories for elliptically shaped molecules [13, 14] predict $\alpha_3/\alpha_2 \approx (b/a)^2$ where *a*,*b* are the major and minor axis lengths of the ellipse; in addition the statistical physics approach of [15] suggests $\alpha_3/\alpha_2 \to 0$ at low temperature—equivalently $(\eta_1-\eta_2)/\gamma_1 \to 1$. These results all imply that $(\eta_1-\eta_2)/\gamma_1 \approx 1$ is a rather fundamental consequence of uniaxial symmetry at low temperatures.

We suggest that to overcome this limitation it may be of interest to examine pi-cell materials of unusual symmetry and molecular packing configurations departing from idealized nematic uniaxiality. In the vicinity of a phase transition to the isotropic the measurements of Graf *et al.* [16] indicate a ratio $(\eta_1-\eta_2)/\gamma_1 \sim 1.8$ for 8OCBP. This agrees with the numerical calculations of [15] which indicate $|\alpha_3/\alpha_2| \sim 1$ close to the isotropic phase. Pretransitional effects close to smectic phases can have a similar effect (e.g. $(\eta_1-\eta_2)/\gamma_1 \sim 0.62$ near to an underlying SmA [16]). It is unclear how far into the nematic phase this pretransitional influence can be made to persist. Injected smectic phases can occur in the phase diagrams of mixtures of polar and non-polar nematics [17] and may offer one approach to utilizing pretransitional phenomena, by formulating nematic mixtures lying close (in concentration) to an injected smectic phase across a wide range of temperature.

5. A high pretilt, negative dielectric anisotropy surface mode

We conclude with a comment relating to the dynamics of positive and negative dielectric anisotropy surface mode devices. We have published elsewhere [18] investigations of a negative $\Delta\varepsilon$, high pretilt ($\theta_p \sim 85°$) device which exhibits an equilibrium bend state at zero volts and a nucleated transition to a splay state at higher voltage. Analogously to the pi-cell, this device exhibits surface switching, although in a splayed state with the central director lying at 0°.

The static electro-optic performance (for example, optical retardation vs. voltage) of a pi-cell and this high pretilt device can in principle be made identical by filling equal thickness cells with, respectively, a positive and a negative $\Delta\varepsilon$ material, with $|\Delta\varepsilon|$ and Δn equal. In addition, the splay and bend elastic constants (K_{11} and K_{33}) of the two materials must have their values interchanged between the 'bend operating' pi-cell and the 'splay operating' high pretilt device. Finally, we require the pretilt of the (low pretilt) pi-cell and the high-pretilt device to be 'opposite' in the sense, $\theta_{\text{pi-cell}} = 90° - \theta_{-\Delta\varepsilon\,\text{device}}$, for identical static behaviour.

For the dynamic characteristics (e.g. response speed) of these two surface mode devices to be identical, a consideration of equations (6) and (7) shows it is also necessary to interchange the values of η_1 and η_2, between the pi-cell material and the material used in the high pretilt device. This is easily understood if we consider the dynamic response of each device immediately following removal of a high voltage. In the case of a pi-cell, director relaxation occurs from an initial high tilt (bend) state towards a lower tilt (bend) state. For the negative $\Delta\varepsilon$ (splay) device, relaxation is in the low-to-high tilt direction. Thus, for equivalent dynamic performance, given a pi-cell material with a viscosity conforming to one of the ($\cos^2\theta$-like) unbroken lines in figure 5 (for example, $\eta_2/\gamma_1 = 0.5$), the negative $\Delta\varepsilon$ device requires a material with a viscosity corresponding to the complementary ($\sin^2\theta$-like) dashed line (e.g. $\eta_2/\gamma_1 = 1.5$). We see that calamitic molecules (for which $\eta_1 > \eta_2$) are naturally suited for use in pi-cells where a low viscosity is required at high angles (practical device operation mostly occurs in a highly tilted bend state), whilst optimization of the performance of the negative $\Delta\varepsilon$ mode ideally requires consideration of $\eta_2 > \eta_1$ (i.e. discotic-type) materials.

Experimentally, we have not found a positive and a negative $\Delta\varepsilon$ material with exactly 'opposite' viscoelastic constants (in the sense defined above) in order to allow a direct comparison of the performance of the two modes. Our experience in observing a range of positive and negative $\Delta\varepsilon$ calamitic mixtures however, does tentatively suggest that higher optical switching speeds can be expected from the pi-cell.

6. Conclusion

The symmetric nature of the pi-cell results in a simplified equation of director motion in a planar geometry. In the limit of $\eta_1/\gamma_1 = 1$, $\eta_2/\gamma_1 = 0$ and $\eta_{12} = 0$ the equations governing director relaxation following removal of an applied voltage, reduce to a simple one-dimensional diffusion equation, with an additional $\cos^2\theta$ term present, deriving from flow symmetry and indicating the role of flow in enhancing pi-cell speed.

Numerical simulations of the full 1D-dynamical equation of motion indicate that η_{12} plays a minor role in pi-cell electro-optic switching, but that speed improvements may result from optimization of the parameter η_2/γ_1 and by seeking materials with $(\eta_1 - \eta_2)/\gamma_1 \neq 1$. Some practical considerations for obtaining suitable materials have been discussed.

We have argued that a high pretilt surface mode device, analogous to the pi-cell but utilizing negative dielectric anisotropy materials, ideally requires consideration of $\eta_2 > \eta_1$ (i.e. discotic-type) materials for optimized performance.

The authors gratefully acknowledge Dr N. J. Mottram (University of Strathclyde, UK), in particular for useful suggestions in connection with the application of the maximum principle.

Appendix I

Relationship between Miesowicz and Leslie notations

$$\eta_1 = \frac{-\alpha_2 + \alpha_4 + \alpha_5}{2}; \qquad \eta_2 = \frac{\alpha_3 + \alpha_4 + \alpha_6}{2}; \qquad \eta_2 = \frac{\alpha_4}{2};$$

$$\eta_{12} = \alpha_1; \qquad \gamma_1 = \alpha_3 - \alpha_2.$$

Additionally, the Parodi [19] relation $\alpha_6 = \alpha_2 + \alpha_3 + \alpha_5$ is useful.

Appendix II

Proof of enhanced relaxation rate for pi-cells with flow included

Consider the two diffusion equations $\theta_t = (1/\cos^2\theta)\theta_{zz}$ (the 'relaxing with flow' case—more generally $\theta_t = (1 - \gamma^*)\theta_{zz}$ with $\gamma^* < 1$ in bulk, the proof is the same) and $\Theta_t = \Theta_{zz}$ ('without flow'). Consider one half ($0 \leqslant z \leqslant 1/2$) of the cell. We have boundary conditions $\theta(0, t) = \Theta(0, t) =$ surface pretilt, $\theta(1/2, t) = \Theta(1/2, t) = \pi/2$, and initial condition $\theta(z, 0) = \Theta(z, 0) = f(z)$. Introduce $H(z, t) = \theta - \Theta$, a measure of the amount by which θ leads (or lags) Θ. We note:

$$H_t - H_{zz}(1/\cos^2\theta) = \Theta_{zz}(1/\cos^2\theta - 1) < 0 \qquad (1)$$

where the inequality follows since the director is relaxing (so $\Theta_t = \Theta_{zz} < 0$), and $\cos^2\theta < 1$ in the bulk. Since this equation for H is uniformly parabolic, H must attain a maximum somewhere (by the 'maximum principle' [20]).

We wish to show $H(z, t) < 0$ in the bulk. Since $H(0, t) = H(1/2, t) = 0$ on the surfaces, we may equivalently show that H is never a maximum in the bulk. Suppose H were to reach a maximum at a point in the bulk. Then, by simple calculus we must have $H_{zz} \leqslant 0$, $H_t = 0$, $H_z = 0$ at that point. But these conditions are incompatible with equation (1). Thus the surface value, $H = 0$, is a maximum and we have proved $\theta < \Theta$ $(t > 0, 0 < z < 1/2)$, which is equivalent to the statement that the presence of flow speeds relaxation. The result follows similarly for the other half-cell.

References

[1] Bos, P. J., and Rickey Koehler/Beran, K., 1984, *Mol. Cryst. liq. Cryst.*, **133,** 329.

[2] Ericksen, J. L., 1960, *Arch. ration. Mech. Anal.*, **4,** 231; Ericksen, J. L., 1966, *Phys. Fluids*, **9,** 1205.

[3] Leslie, F. M., 1966, *Q. J. Mech. appl. Math.*, **19,** 357; Leslie, F. M., 1968, *Arch. ration. Mech. Anal.*, **28,** 265.

[4] Miesowicz, M., 1946, *Nature*, **158,** 27.

[5] De Jeu, W. H., 1980, *Physical Properties of Liquid Crystalline Materials* (Gordon and Breach).

[6] Van Doorn, C. Z., 1975, *J. appl. Phys.*, **46,** 3738.

[7] Berreman, D. W., 1975, *J. appl. Phys.*, **46,** 3746.

[8] Brochard, F., and de Gennes, P. G., 1970, *J. Phys. (Paris)*, **31,** 691.

[9] Gahwiller, Ch., 1973, *Mol. Cryst. liq. Cryst.*, **20,** 301.

[10] Armitage, D., and Larimer, J., 1996, *SID 96 Dig.*, 584.

[11] Blinov, L. M., and Chigrinov, V. G., 1994, *Electro-optic Effects in Liquid Crystal Materials* (Springer Verlag), Chap. 2.

[12] Kneppe, H., Schneider, F., and Sharma, N. K., 1982, *J. chem. Phys.*, **77,** 3203.

[13] Helfrich, W., 1969, *J. chem. Phys.*, **51,** 4092.

[14] Ehrentraut, H., and Hess, H., 1995, *Phys. Rev. E*, **51,** 2203.

[15] Osipov, M. A., and Terentjev, E. M., 1989, *Phys. Lett. A*, **134,** 301.

[16] Graf, H.-H., Kneppe, H., and Schneider, F., 1981, *Mol. Phys.*, **65,** 23.

[17] Bradshaw, M. J., and Raynes, E. P., 1983, *Mol. Cryst. liq. Cryst.*, **99,** 107.

[18] Walton, H. G., and Towler, M. J., 2000, *Liq. Cryst.*, **27,** 157.

[19] Parodi, O., 1970, *J. Phys.*, **31,** 581.

[20] Du Chateau, P., and Zachmann, D. W., 1986, *Schaum's Outline of Partial Differential Equations* (McGraw-Hill), Chap. 4.